Lines and planes

A line in 3 space can be given by vector equation of the form $L(t) = \vec{p} + t\vec{v}$

where p is position vector and v is the direction of the line

Ex: compute the vector equation of the line through $(-6,2,3)$ and parallel to $m(t) = \langle 6t-3, 5t+1, -2-3t \rangle$

$\vec{p} = \langle -6, 2, 3 \rangle$

The given line has vector equation

$m(t) = (-3, 1, -2) + t\underbrace{(6, 5, -3)}_{\vec{v}}$

$$L(t) = \langle -6, 2, 3 \rangle + t\langle 6, 5, -3 \rangle$$

The parametric equations of a line have a form

$$\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$$

For $m(t) = \langle 6t-3, 5t+1, -2-3t \rangle$: $\begin{cases} x(t) = 6t-3 \\ y(t) = 5t+1 \\ z(t) = -2-3t \end{cases}$

For $L(t) = \langle -6, 2, 3 \rangle + t\langle 6, 5, -3 \rangle = \langle -6+6t, 2+5t, 3-3t \rangle$

$$\begin{cases} x(t) = -6+6t \\ y(t) = 2+5t \\ z(t) = 3-3t \end{cases}$$

The symmetric ~~form~~ equations of a line the form

$$\frac{x-x_0}{t} = \frac{y-y_0}{t} = \frac{z-z_0}{t}$$

Ex: Compute symmetric form

$$\begin{cases} x(t) = 6t - 4 \\ y(t) = 5t + 2 \\ z(t) = -3t + 3 \end{cases} \rightarrow \quad t = \frac{x+4}{6} \quad t = \frac{y-2}{5} \quad t = \frac{z-3}{-3}$$

$$\frac{x+4}{6} = \frac{y-2}{5} = \frac{z-3}{-3}$$

Terminology

1. ~~lines~~ parallel when they have the same direction
2. intersecting when they have a common ~~direction~~ point
3. Skew when they are neither parallel nor intersecting

Ex: Do the lines $L_1 = \langle 3,4,1 \rangle + t\langle 2,-1,3 \rangle$ and $L_2 = \langle 1,3,4 \rangle + t\langle 4,-2,5 \rangle$ parallel, intersect, skew?

- Lines are not parallel since the directions vectors are not scalar multiples of each other

$$\langle 2,-1,3 \rangle \neq \langle 4,-2,5 \rangle$$

- Lines intersect if $L_1(t) = L_2(s)$ ~~M~~ t and s can be different and still intersect

$$\langle 3+2t, 4-t, 1+3t \rangle = \langle 1-4s, 3-2s, 4+5s \rangle$$

$$\begin{cases} 3+2t = 1-4s \\ 4-t = 3-2s \\ 1+3t = 4+5s \end{cases} \rightarrow \quad \begin{aligned} 2t+4s &= -2 \\ -t+2s &= -1 \\ 3t-5s &= 3 \end{aligned}$$

Solve system of equations

Lines do not intersect

- Since lines are not parallel and do not intersect they must be skewed

Last time: vector equation of a plane

$$\vec{n} \cdot (\vec{x} - \vec{p}) = \vec{0}$$

$\vec{n}$: normal vector; $\vec{x}$: vector of variables; $\vec{p}$: position vector

Ex: compute the plane through $(3,1,4)$ the line of intersection of planes $x+2y+3z=1$ and $2x-y-z=2$

Sol: we are given $\vec{p} = \langle 3,1,4 \rangle$

$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \langle 1,2,3 \rangle \times \langle 2,-1,-1 \rangle = \langle 1, 7, -5 \rangle$

$$\begin{cases} x+2y+3z=1 \\ 2x-y-z=2 \end{cases} \xrightarrow{\text{solve system}}$$

$$\begin{cases} x = 1 - \frac{1}{5}t \\ y = -\frac{7}{5}t \\ z = t \end{cases} \rightarrow \quad \vec{u} = \langle 2, 1, 4 \rangle$$

$\vec{n} = \vec{u} \times \vec{v} = \langle -33, 14, 13 \rangle$

$\langle -33, 14, 13 \rangle \cdot \langle x-3, y-1, z-4 \rangle$

12.6 Quadratic surfaces

IDea: given a degree 2 poly in 3-space what does it solution set look like

$P(x,y,z) = x^2 - y$ "degenerate"

non-degenerate: Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Cone $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$

Eliptic paraboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z}{c} = 0$